International Journal of Education and Science Research Review
ISSN 2348-6457
www.ijesrr.org April- 2017, Volume-4, Issue-2 Email- editor@ijesrr.org

INVENTORY MODEL FOR PERISHABLE ITEMS AMONG INFLATION INDUCED DEMAND BELOW CREDIT PHASE

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ABSTARCT

In the classical inventory models payment for the items paid by the supplier depends on the payment paid by the retailer and in such cases the supplier offers a fixed credit phase to the retailer. During which no interest will be charged by the supplier so there is no need to pay the purchasing cost by the retailer. After this credit phase up to the end of a phase interest charged and paid by the retailer. In such situations the retailer starts to accumulate revenue on his sale and earn interest on his revenue. If the revenue earned by the retailer up to the end of credit phase is enough to pay the purchasing cost or there is a budget, the balance is settled and the supplier does not charge any interest, otherwise the supplier charges interest for unpaid balance after the credit phase. The interest and the remaining payment are made at the end of replenishment cycle. Deterioration is defined as decay, damage or spoilage. Food items, photographic films, drugs, chemicals, electronic components and radioactive substances are some examples of items in which sufficient deterioration may occur during the normal storage phase of the units and consequently this loss must be taken into account while analyzing the inventory system.

Key words: Deterioration, spoilage model

INTRODUCTION

In traditional EOQ models the payment time does not affect the profit and replenishment policy. If we consider the inflation then order quantity and payment time can influence both the supplier’s and retailer’s decisions. A large pile of perishable foods such as fruits, vegetables, milk, bread, chocklet etc. attracts the consumers to buy more. Buzacott [1] considered an EOQ model with different type of pricing policies under inflation. Baker and Urban [2] proposed a deterministic inventory model for deteriorating items with stock level dependent demand rate. Mandal and Phaujdar [3] presented an inventory model for deteriorating items with stock level dependent consumption rate. Vrat and Padmanabhan developed two inventory models [4] and [5]. The model [4] is an inventory model with stock dependent consumption rate under inflation and the model [5] is an EOQ model for perishable products with stock dependent selling rate. Bose et al. [6] presented an EOQ model for deteriorating items with linear time dependent demand and shortages. They also considered the concept of inflation and time discounting in their inventory model. Mandal and Maiti [7] proposed an inventory model for damageable products with stock dependent demand and variable replenishment rate. Chung and Lin [8] determined an optimal replenishment policy for an inventory model of deteriorating items with time discounting. Chang [9] proposed an EOQ model for deteriorating items under inflation and time discounting. He assumed that the supplier offers a trade credit policy to the retailer, when the retailer’s order size is larger than a certain level. Dye and Ouyang [10] developed an EOQ model for perishable items with stock dependent selling rate by allowing shortages. Hou [11] presented an inventory model for deteriorating items with stock dependent consumption rate and shortages. He also considered the effect of inflation and time discounting in his inventory model. Jaggi et al. [12] determined an optimal ordering policy for an inventory model of deteriorating items with time dependent demand. They also introduced the concept of inflation in their inventory model. Sana and Chaudhuri [13] developed a deterministic EOQ model for deteriorating items with stock dependent demand and permissible delay in payments. Valliaththal and Uthayakumar [14] presented an EOQ model for perishable products with stock dependent selling rate and shortages. Roy et al. [15] developed an inventory model for deteriorating items with stock dependent demand. They also considered the fuzzy inflation rate and time discounting over a random planning horizon. Sana developed two inventory models [16] and [21]. The model [16] is a lot size inventory model with time varying deterioration rate and stock dependent demand by allowing shortages. And in the model [21] she considered a control policy for a production system with stock dependent demand. Chang et al. [17] determined an optimal replenishment policy for an inventory model of non-instantaneous deteriorating items with stock dependent demand. Sarkar et al. [18] presented an EMQ (economic manufacturing quantity) model for imperfect production process. They also considered the time dependent demand and time value of money under inflation.

ASSUMPTION AND NOTATIONS

The following assumptions are used to develop a foresaid model:

- Shortages are allowed
- If the retailer pays by M. then the supplier does not charge to the retailer. If the retailer pays after M and before N (N>M), he can keep the difference in the unit sale price and unit purchase price in an interest bearing account at the rate of Ic/Unit/Year. During [M,N], the supplier charges the retailer an interest rate of IC_1/Unit/Year on unpaid balance. If the retailer pays after N, then supplier charges the retailer an interest rate of IC_2/Unit/Year ($IC_1 > IC_2$) on unpaid balance.

The notations are as follows

- s = selling price /unit
- C_0 = the unit purchase cost with $C_0 < s$
- M = the first offered credit phase in selling the account without any charges,
- N = the second permissible credit phase in settling the account with interest charge IC_2 on unpaid balance and N>M
- IC_1 = the interest charged per $ in stock per year by the supplier when retailer pays during [M,N]
- IC_2 = the interest charged per $ in stock per year by the supplier when retailer pays during [N,T]($IC_1 > IC_2$)
- Ie = the interest earned / $ / year
- r = discount rate $r > \alpha$
- IE = the interest earned / time unit
- IC = the interest charged /time unit
- T = length of replenishment cycle.
- The demand rate is exponentially increasing and $D(t)=\lambda_0 e^{\alpha t}$ where $0 \le \alpha \le 1$ is a constant inflation rate and λ_0 is initial demand rate.
- A_0 = ordering cost / order
- C_{10} = carrying cost / unit time
- C_{20} = shortage cost / unit time
- θt = variable deterioration rate
- A discounted cash flow (DCF) approach is used to consider the various costs at various times $(r > \alpha)$ is discount rate.
- L is the length of finite planning horizon.

MATHEMATICAL FORMULATION

Assuming continuous compounding of inflation, the ordering cost, unit cost of the item, out of pocket inventory carrying cost and storage cost at any time t are

$$A(t) = A_0 e^{\alpha t}$$

$$C(t) = C_0 e^{\alpha t}$$

$$C_1(t) = C_{10} e^{\alpha t}$$

and

$$C_2(t) = C_{20} e^{\alpha t}$$

the planning horizon L has been discarded into n equal cycles of length T(i.e. T= L/n) let us consider the ith cycle i.e. $t_{i-1} \le t \le t_i$ where $t_0 = 0$, $t_n = L$, $t_i - t_{i-1} = T$ and t_i=it (i = 1,2,.......n). At the beginning of ith cycle a batch of q_i units enters the inventory system from which s_i units are delivered towards backorders leaving a balance of I_{0i} units as the initial inventory level of ith cycle $q_i = I_{0i} + s_i$. there after as time passes, the inventory level gradually decreasing mainly due to demand and partially due to deterioration and reaches zero at time t_{i1} (Fig.1) further demands during the remaining phase of the cycle from t_{i1} to t_i are backlogged and are of fulfilled by a new procurement.

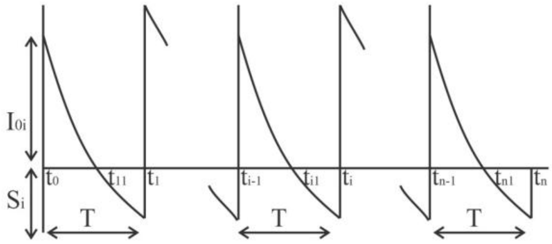


Fig. 1

Now $t_{i1} = t_i - kt = (i-k)L/n\, i = (1,2,......n)(0 \le k \le 1)$ where kt is the fraction of the cycle having shortages. Let $I_i(t)$ be the inventory level of the ith cycle at time $t(t_{i-1} \le t \le t_i, i = 1,2,......n)$. Now at the beginning of each cycle there will be cash out flow of ordering cost and purchase cost. Further since the inventory carrying cost is proportional to the value of the inventory, the out of pocket (Physical storage) inventory carrying cost per unit time at time t is $I(t)C_1(t)$. Similarly the shortage cost can also be obtained. The inventory level is represented by the following differential equations:

$$\frac{dI_i(t)}{dt} + \theta t I_i(t) = -\lambda(t) = -\lambda_0 e^{\alpha t} \qquad t_{i-1} \le t \le t_{i1} \quad i = 1, 2, ...n \qquad .....(1)$$

$$\frac{dI_i(t)}{dt} = -\lambda(t) = -\lambda_0 e^{\alpha t} \qquad t_{i1} \le t \le t_i \quad i = 1, 2, ...n \qquad ...... (2)$$

The solution of the above differential equation along with the boundary condition $I(t_{i-1}) = I_{0i}$ and $I_i(t_{i1}) = 0$ is

$$I_i(t) = I_{0i}\, e^{\theta/2 (t_{i-1}^2 - t^2)} + \lambda_0 \left[(t_{i-1} - t) + \frac{\alpha}{2}(t_{i-1}^2 - t^2) + \frac{(\theta + \alpha^2)}{6}(t_{i-1}^3 - t^3) \right] e^{-\theta t^2/2} \qquad .... (3)$$

Now put $I_i(t_{i1}) = 0$ in (3a) then

$$I_{0i} = -\lambda_0 \left[(t_{i-1} - t_{i1}) + \frac{\alpha}{2}(t_{i-1}^2 - t_{i1}^2) + \frac{(\theta + \alpha^2)}{6}(t_{i-1}^3 + t_{i1}^3) \right] e^{-\theta t_{i-1}^2} \quad i = 12, ...n \qquad ... (4)$$

Now put $I_i(t_i) = -s_i$ in (3b) then

$$s_i = \frac{\lambda_0}{\alpha}\left(e^{\alpha t_i} - e^{\alpha t_{i1}}\right) \qquad i = 12, ...n \qquad .... (5)$$

Now we put the value of I_{0i} in (3a) then

$$I_i(t) = \lambda_0 e^{-\theta t^2/2} \left[(t_{i1} - t) + \frac{\alpha}{2}(t_{i1}^2 - t^2) + \frac{(\theta + \alpha^2)}{6}(t_{i1}^3 - t^3) \right] \quad t_{i-1} \le t \le t_{i1} \quad i = 1, 2, \qquad ...(6)$$

Further batch size qi for the ith cycles is :

$$q_i = I_{0i} + s_i$$

$$q_i = -\lambda_0 \left[(t_{i-1} - t_{i1}) + \frac{\alpha}{2}(t_{i-1}^2 - t_{i1}^2) + \frac{(\theta + \alpha^2)}{6}(t_{i-1}^3 - t_{i1}^3) \right] \times e^{-\theta/2\, t_{i-1}^2} + \frac{\lambda_0}{\alpha}\left(e^{\alpha t_i} - e^{\alpha t_{i1}}\right) \quad i = 1, 2, ...n$$

... (7)

(1) Present worth of ordering cost for the ith cycle A_i is –

$$A_i = A(t_{i-1})e^{-rt_{i-1}} = A_0\, e^{(\alpha-r)t_{i-1}} \quad i=1,2,.....................n \qquad ....(8)$$

(2) Present worth of the purchase cost for the ith cycle P_i is -

$$P_i = q_i C(t_{i-1})e^{-rt_{i-1}} = q_i C_0\; e^{(\alpha-r)t_{i-1}} \quad i=1,2,.....................n \qquad ...(9)$$

(3) Present worth of the inventory carrying cost for the ith cycle H_i is

$$H_i = C_1 (t_{i-1})e^{-rt_{i-1}} \int_{t_{i-1}}^{t_{i1}} I_i(t)e^{-rt}dt$$

$$H_i = C_{10}\, \lambda_0 e^{(\alpha-r)t_{i-1}} \int_{t_{i-1}}^{t_{i1}} e^{-\theta t^2/2} \left[(t_{i1}-t) + \frac{\alpha}{2}(t_{i1}{}^2 - t^2) + \frac{(\theta+\alpha^2)}{6}(t_{i1}{}^3 - t^3)\right] e^{-rt}dt \qquad ...(10)$$

(4) Present worth of the shortage cost for the ith cycle is –

$$\pi_i = C_2 (t_{i-1})e^{-rt_{i-1}} \int_{t_{i1}}^{t_i} I_i(t)e^{-rt}dt$$

$$= C_{20}\, e^{(\alpha-r)t_{i-1}} \frac{\lambda_0}{\alpha} \int_{t_{i1}}^{t_i} (e^{\alpha t} - e^{-\alpha t_{i1}})e^{-rt}dt \qquad .....(11)$$

$$= \lambda_0 C_{20} \left[\frac{e^{(\alpha-r)t_i} - e^{(\alpha-r)t_{i1}}}{(\alpha-r)} + \frac{e^{\alpha t_i}}{r}(e^{-rt_i} - e^{-rt_{i1}})\right] \times e^{(\alpha-r)t_{i-1}} \quad i=1,2,........n$$

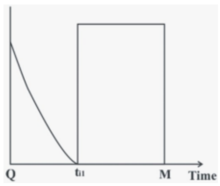
Therefore the present worth of the total variable cost for the ith cycle Pw_i is the sum of the ordering cost A_i purchase cost P_i, inventing carrying cost (H_i) and shortage cost (π_i) i.e.

$$Pw_i = A_i + P_i + H_i + \pi_i \qquad ...(12)$$

The present worth of the total variable cost of the system during the entire time horizon L is given by –

$$PW_L(k,n) = \sum_{i=1}^{n} PW_i = \sum_{i=1}^{n} (A_i + P_i + H_i + \pi_i) \qquad ...(13)$$

Case I M $\geq$ t_{i1}



$M \geq t_{i1}$

Inventory level Fig. 2

In the first case, retailer does not pay any interest to the supplier. Here retailer sells I_s units during (0, t_{i1}) time internal and paying for CI_s units in full to the supplier at time $M \geq t_{i1}$so interest charges are zero i.e.

$$IC_1 = 0 \qquad ..(14)$$

Retailers deposits the revenue in an interest bearing account at the rate of Ie/$/year.

Therefore, interest earned IE_1, per year is

$$IE_1 = \frac{sI_e}{T_2}\left[\int_0^{t_{i1}} D(t)t\,dt + (M - t_{i1})\int_0^{t_{i1}} D(t)\,dt\right] \qquad ....(15)$$

Total cost per unit time of an inventory system is –

$$T[PW_L(k,n)] = \sum_{i=1}^{n} PWi + IC_1 - IE_1$$

$$= \sum_{i=1}^{n} (A_i + P_i + H_i + \pi_i) + IC_1 - IE_1 \qquad ...(16)$$

Case II – M < t_{i1}<N

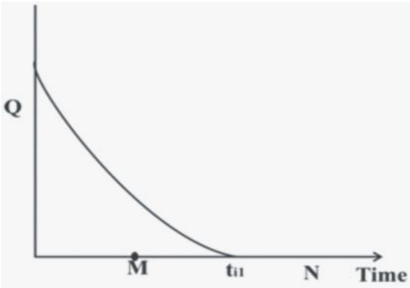


Fig 3 M<ti1<N

In the second case, supplier charges interest at the rate IC_1 on unpaid balance –
Interest earned, IE_2 during [0,M] is

$$IE_2 = sIe\int_0^M D(t)t\,dt \qquad \ldots (17)$$

Retailer pay Is units purchased at time t = 0 at the rate of C/$/unit to the supplier during [0,M]. The retailer sells D(M).M units at selling price s/unit.
So, he has generated revenue of s D(M).M + IE_2.
Then two sub cases may be arises.

Sub Case 2.1 –
Let sD(M).M + $IE_2 \geq$ CIs retailer has enough money to settle, his account for all Is units procured at time t =0 then interest charge will be

$$IC_{2.1} = 0 \qquad \ldots (18)$$

and interest earned

$$IE_{2.1} = \frac{IE_2}{T_2} \qquad \ldots (19)$$

So the total cost $T_{2.1}[PW_L(k,n)]$ per unit time of inventory system is

$$T_{2.1}\left[PW_L(k,n)\right] = \sum_{L=1}^{n}(A_i + P_i + H_i + \pi_i) + IC_{2.1} - IE_{2.1} \qquad \ldots (20)$$

Sub Case 2.2 –
Let sD(M).M + IE_2 < CIs here retailer will have to pay interest on unpaid balance U_1= CIs – (sD(M).M + IE_2) at the rate of IC_1 at time M to the supplier. Then interest paid per unit time us given by –

$$IC_{2.2} = \frac{U_1^2 IC_1}{I_s}\int_M^{t_i} I_i(t)\,dt \qquad .. (21)$$

and interest earned

$$IE_{2.2} = \frac{IE_2}{T_2} \qquad \ldots (22)$$

So the total cost $T_{2.2}[PW_L(k,n)]$ per unit time of inventory system is

$$T_{2.2}\left[PW_L(k,n)\right] = \sum_{L=1}^{n}(A_i + P_i + H_i + \pi_i) + IC_{2.2} - IE_{2.2} \qquad \ldots (23)$$

Table 1: Retailer does not pay any interest to the Supplier

N	T_2	t_1	TC
1	0.754546	0.116863	1284.561
2	0.786092	0.118116	1289.345
3	0.806756	0.118522	1292.717
4	0.836491	0.118723	1331.382
5	0.865356	0.118844	1348.485

Table 2: Supplier charges interest but Retailer has enough money to settle his account

N	T_2	t_1	TC
1	0.764371	0.124818	1615.17
2	0.787892	0.125711	1531.16
3	0.815642	0.125993	1518.25
4	0.826030	0.126018	1487.83
5	0.857485	0.126614	1416.19

CONCLUSION

Most products experience a phase of rapid demand increase during the introduction phase of product life cycle, level off in demand after reaching their maturity phase, and will enter a phase of sales decline due to new competing products or changes in consumer preference. An inventory control is an intriguing yet practicable issue of decision science when inflation induced demand is involved. The effect of inflation on an inventory system has been taken into consideration. Cost minimization technique is used to get the expressions for total cost and other parameters.

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